Variants of preconditioned conjugate gradient methods applied to linear systems arising from interior point methods

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Interior Point Methods

*The Newton direction $(\Delta x, \Delta y, \Delta z)$ is computed by solving the system of linear equations:

$$\begin{bmatrix} A & 0 & 0 \\ 0 & A^T & I \\ Z & 0 & X \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix} = \begin{bmatrix} r_p \\ r_d \\ r_a \end{bmatrix}$$

*The system can be reduced to a positive-definite system called normal equations:

$$(ADA^T)\Delta y = r_p + A(Dr_d - Z^{-1}r_a)$$

with $D = Z^{-1}X$

Interior Point Methods

Normal Equations can be solved by

*Direct methods: Cholesky Factorization.

*Iterative methods: Preconditioned Conjugate Gradient methods.

Interior Point Methods

$$(ADA^T)\Delta y = r_p + A(Dr_d - Z^{-1}r_a)$$

CGNR: Conjugate Gradient Normal Residual

$$G^T Gu = G^T d$$

where:

$$G = D^{\frac{1}{2}}A^T$$

$$u = \Delta y$$

$$G^T d = AD^{\frac{1}{2}}D^{\frac{1}{2}}\left(D^{-1}\tilde{b} - D^{-1}x + r_d - D^{-1}Z^{-1}r_a\right)$$

$$\tilde{b} = A^T(AA^T)^{-1}b$$

Interior Point Methods

$$(ADA^T)\Delta y = r_p + A(Dr_d - Z^{-1}r_a)$$

CGNE: Conjugate Gradient Normal Error

$$GG^T u = d$$

where:

$$G = AD^{\frac{1}{2}}$$

$$u = \Delta y$$

$$d = r_p + A(Dr_d - Z^{-1}r_a)$$

Numerical Experiments

* The new versions of PCG was integrated to Modified-PCx code.

* Modified-PCx uses The Classic Preconditioned Conjugate Gradient methods for solve normal equation with a Hybrid-preconditioned.

* All test problems are public domain linear problems : STOCHLP, QAP, NETLIB, MISC.

Computational Results

*The performance of the two new PCG versions were compared with the classic one.

*For comparisons were used:

  *Interior point outer Iterations;

 *Total Preconditioner CG inner iterations;

  *Execution time

PCG vs PCGNR

Problems	Iterations PCG (IPM)	Iterations PCGNR(IPM)	Time PCG	Time PCGNR
scsd8-2c-64	382 (7)	5864 (10)	4,04	20,00
scsd8-2r-432	6519 (18)	11397 (22)	39,28	180,96
els19	26294 (31)	-	168,18	-
chr22b	25580 (29)	-	71,46	-
scr15	15973 (24)	-	28,81	-
scr20	36360 (21)	-	223,70	-
rou20	54326 (24)	-	3216,66	-
ste36a	312988 (38)	-	32183,05	-
ste36b	-	-	-	-
stocfor3	69507 (32)	-	338,54	-
qap12	29962 (20)	-	285,54	-
qap15	113695 (24)	-	4844,20	-
nug12	23677 (20)	-	245,79	-
nug15	91134 (23)	-	4466,58	-
pds-10	6872 (47)	8851 (49)	63,36	77,26
pds-20	52372 (60)	-	741,67	-
pds-30	31129 (73)	70946 (79)	741,93	1748,39
pds-40	43516 (79)	127153 (84)	1463,64	4813,55
pds-50	71481 (79)	-	2905,05	-
pds-60	83095 (85)	-	4177,60	-
pds-70	83050 (84)	-	5052,97	-
pds-80	80742 (83)	-	5729,80	-
pds-90	97909 (82)	-	7467,40	-
pds-100	122237 (86)	-	9829,66	-

PCG vs PCGNE

Problems	Iterations PCG (IPM)	Iterations PCGNE(IPM)	Time PCG	Time PCGNE
scsd8-2c-64	382 (7)	389 (7)	4,04	4,03
Scsd8-2r-432	6519(18)	6914(18)	39,28	42,26
els19	26294 (31)	26315 (31)	168,18	172,48
chr22b	25580 (29)	25688 (29)	71,46	72,98
scr15	15973 (24)	15886 (24)	28,81	28,99
scr20	36360 (21)	35789 (21)	223,70	254,20
rou20	54326 (24)	54606 (24)	3216,66	3156,59
ste36a	312988 (38)	305066 (38)	32183,05	32256,71
ste36b	-	375810 (37)	-	41786,72
stocfor3	69507 (32)	69828 (32)	338,54	345,72
qap12	29962 (20)	30519 (20)	285,54	305,59
qap15	113695 (24)	92876 (23)	4844,20	4160,35
nug12	23677 (20)	24111 (20)	245,79	249,23
nug15	91134 (23)	-	4466,58	-
pds-10	6872 (47)	6765 (47)	62,94	63,01
pds-20	52372 (60)	53759 (60)	741,67	782,26
pds-30	31769 (74)	31324 (74)	793,66	824,34
pds-40	43516 (79)	43797 (79)	1463,64	1537,23
pds-50	71481 (79)	69471 (79)	2905,05	2959,64
pds-60	83095 (85)	82774 (85)	4177,60	4337,98
pds-70	83050 (84)	77579 (85)	5052,97	4994,45
pds-80	80742 (83)	75626 (83)	5729,80	5658,51
pds-90	97909 (82)	82992 (81)	7467,40	6841,65
pds-100	122237 (86)	126769 (87)	9829,66	10493,45

# Performance Profile
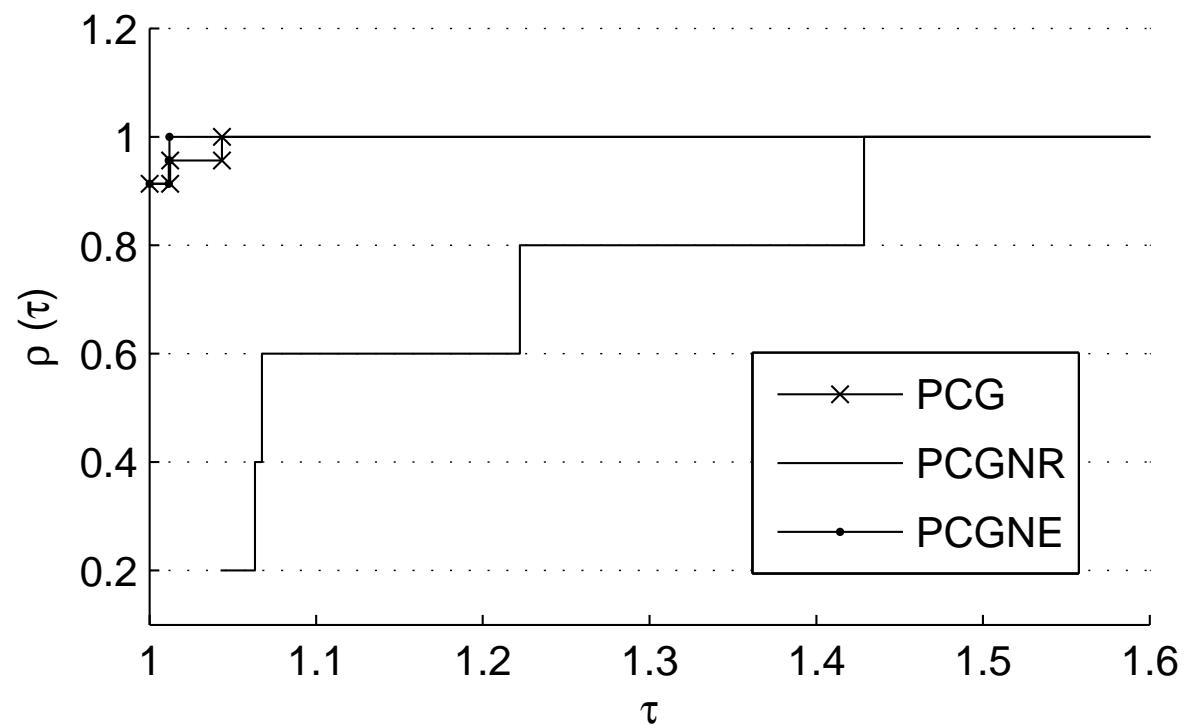
Interior Point outer Iterations

Performance Profile
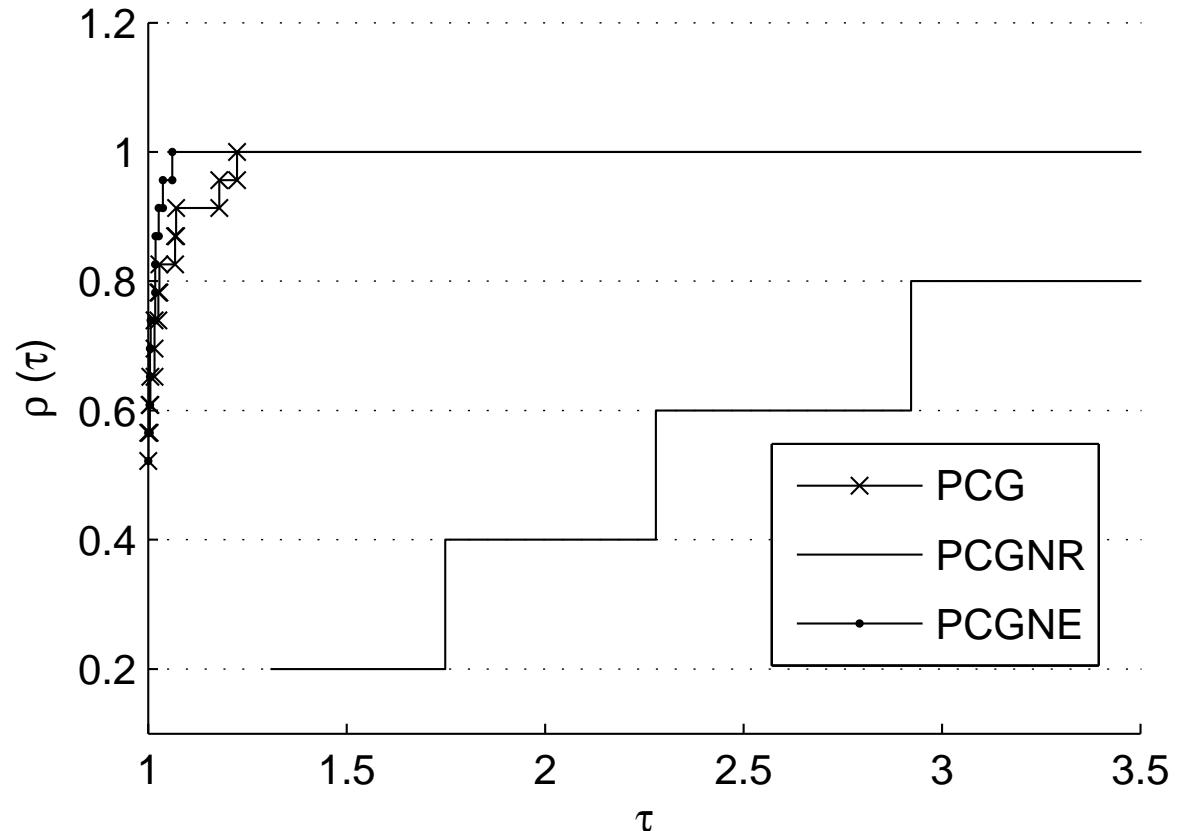
Total Preconditioner CG inner iterations

Performance Profile
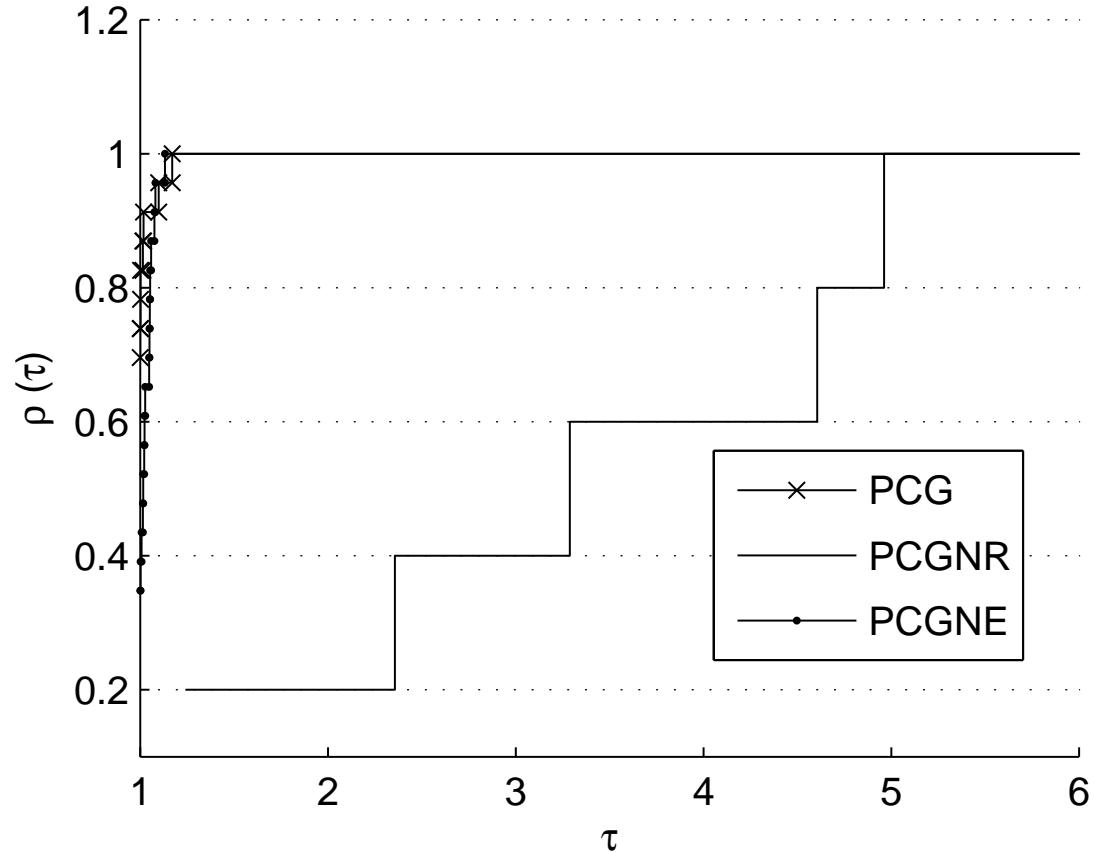
Execution Time

Conclusions

* Two preconditioned conjugate gradient versions for normal system:

    PCGNR

    PCGNE

* The results for classic version of PCG and PCGNE were competitive

* The PCGNR was not appropriate for this system.